

# The Graph of our Mind: graph-theoretic analysis of the human brain

Vince Grolmusz

PIT Bioinformatics Group

Mathematical Institute, Eötvös University

<https://pitgroup.org>

<http://grolmusz.pitgroup.org>

# Quantitativity and mathematics

Physics, in its famous revolution at the beginning of XX<sup>th</sup> century, extensively applied the advanced mathematical tools developed for that time (differential equations, operator theory, probability theory and statistics) together with the revolutionary laboratory tools of physics.

Today, in brain science, the well developed measurement techniques are widely present (imaging techniques, electrophysiology, cell biology techniques), but the input of the mathematics to brain science is mostly missing.

# Graph theory, more exactly, brain graphs could be the rich structure



**Connectome, or brain graph:**

**Nodes (or vertices):** Neurons

**Edges:** two neurons are connected by an edge if there is a synapse (connection) between them.

**Size counts:** ~80 billion neurons in human brain – cannot be handled.

**Lack of measurement methods:** cannot record that graph, no methods.

**Only the whole *C. elegans* „brain” is mapped with several hundred neurons. Even the drosophila brain with 100,000 neurons (the FlyBrain) is still unmapped in its entirety, while enormous resources were spent for this goal (e.g., HHMI Janelia Farm).**

# Brain Graphs from Diffusion MRI

- With diffusion MRI the movement of the water molecules can be tracked : this movement is faster and more probable in the direction of the axons compared to the direction of the cell membrane.
- This way bundles of axons can be observed & recorded



# Not a neuronal-scale graph

Vertices: larger areas of the brain,  
around 1-1.5 cm<sup>2</sup>

Edges: bundles of axons that connect these larger areas of the brain.

Why are these **graphs** are interesting?

- Because the particular shape of the axons that connects two brain areas are indifferent in most cases: the connection counts,
- Because graph theory was born in 1741 by an article of Euler on Königsberg bridges, and in the second half of the XXth century it was developed *enormously*, it has a very rich literature today (do not confuse with the so called „*network science*“).

# Our results in the analysis of the human brain graph

**Data Source:** high quality diffusion MRI images from the US-based large Human Connectome Project, of 500 subjects of age 22-35.

## **Result: Strong sex differences**

Women's brain graph or connectome is much better „connected” than those of men.

Women's brain graph has

- More edges,
- Larger bipartition width
- Larger minimum vertex cover
- Larger eigengap than those of men.

# Our results in the analysis of the graph of our mind

- **Sex differences are not artifacts:**

Statistically, women have smaller brains than men; it may happen that some artifact implies these differences, and they are not due to sex, but rather to size differences.

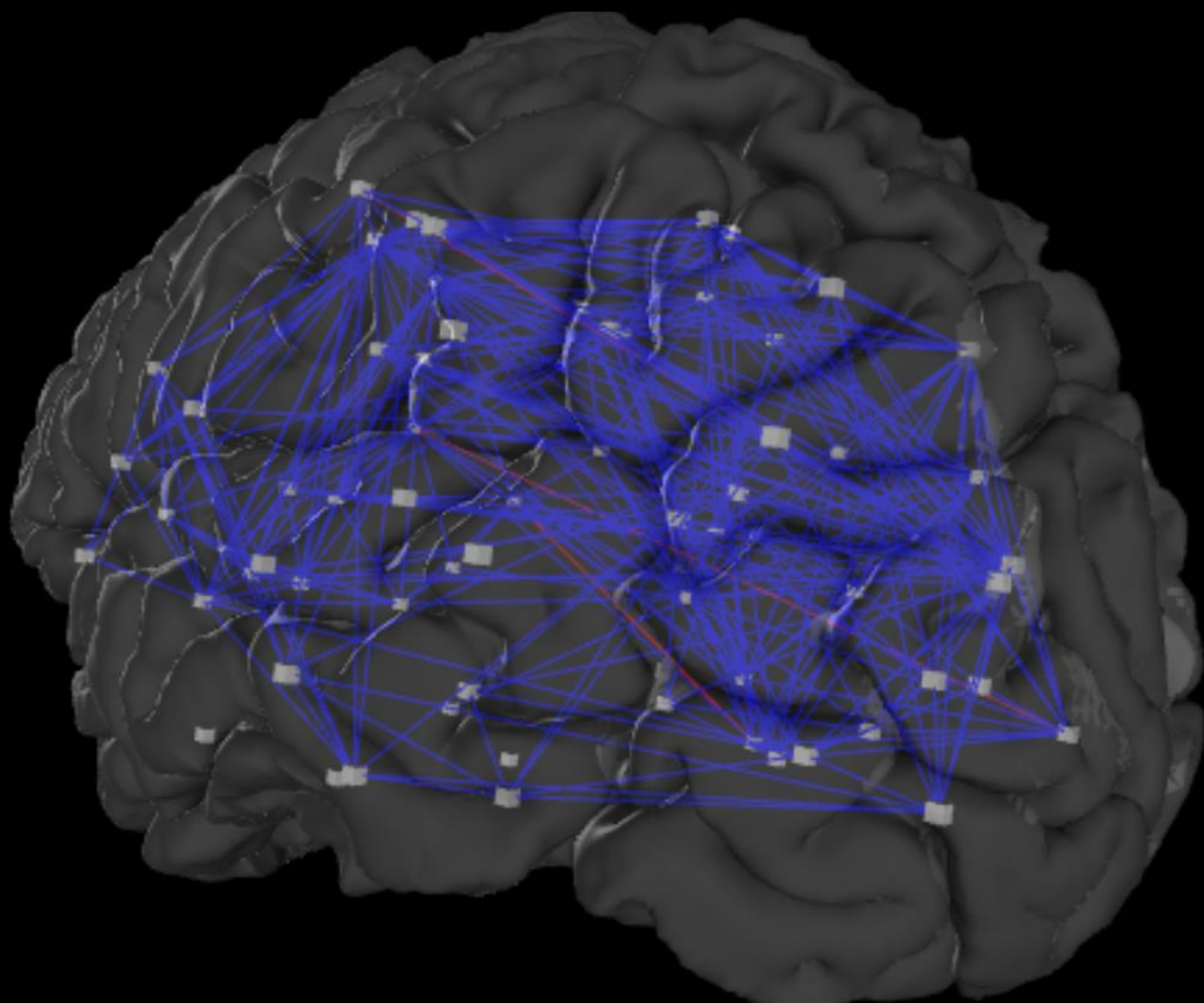
**Not artifact:** We have chosen 36 small-brain men and 36 large-brain women s.t. each woman has larger brain in the group than all of the men;

- The advantage of the women remained valid

# The „Budapest Reference Connectome” Server

- 477 subjects, 477 brain graphs on the very same, anatomically identified vertex set.
- The server draws graph edges that are present in at least  $k$  subjects' graphs, from  $k=1$  through 477.
- That is, one can find connections, or the edges that are general enough in humans.
- Sort of consensus brain graph.
- <https://connectome.pitgroup.org>



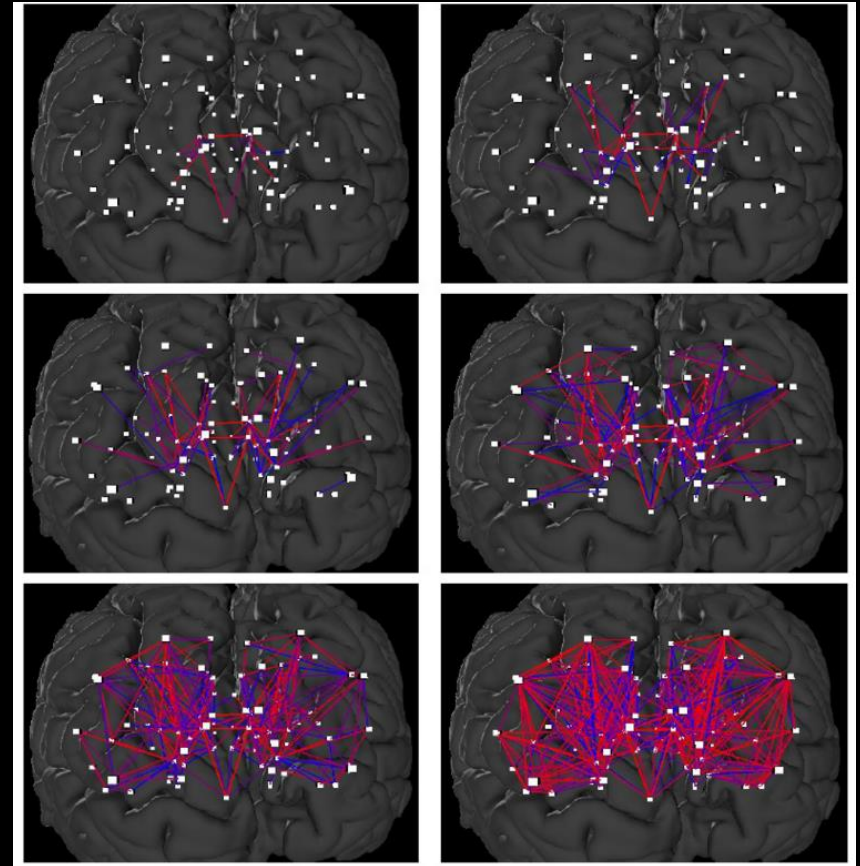


# The Consensus Connectome Dynamics

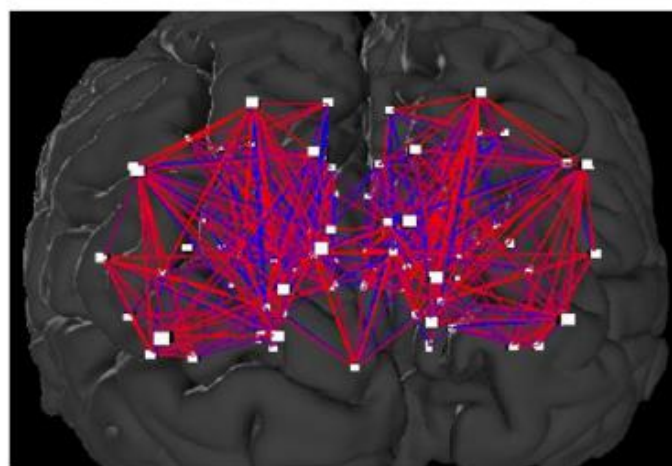
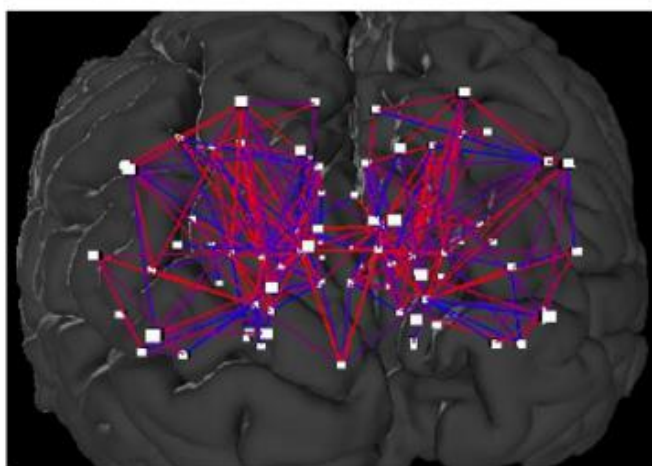
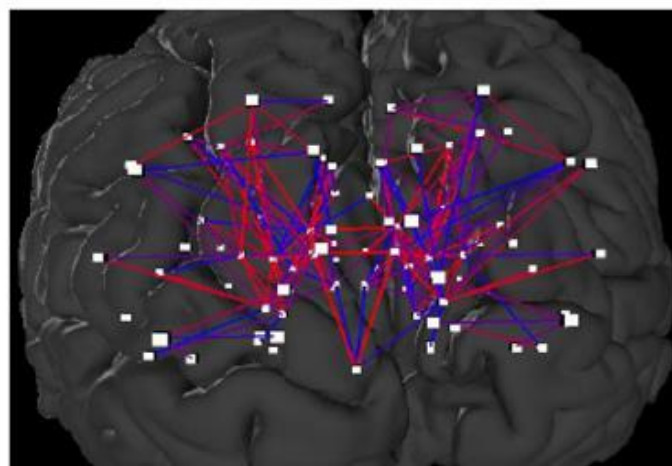
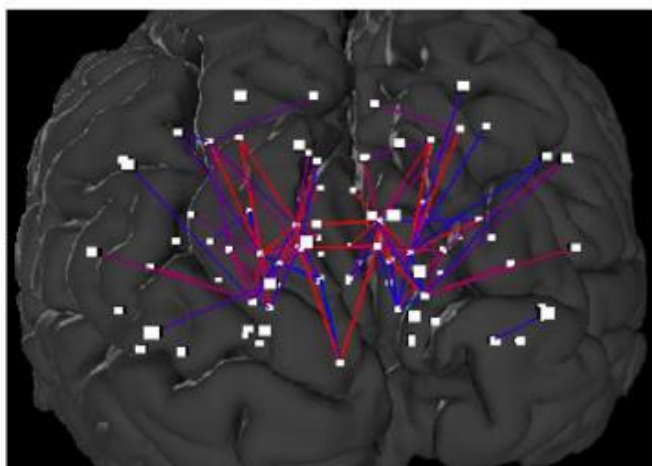
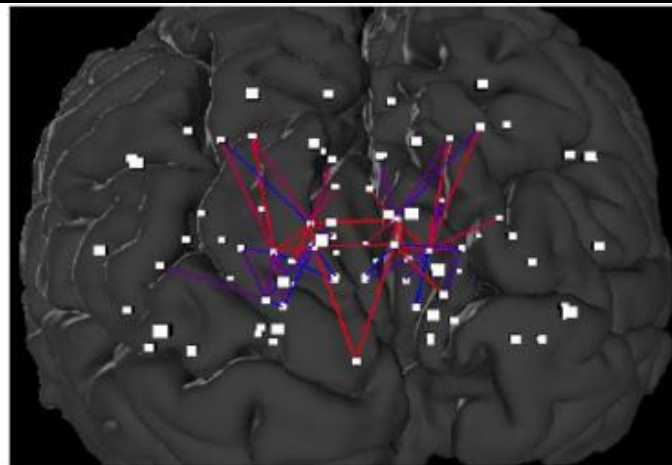
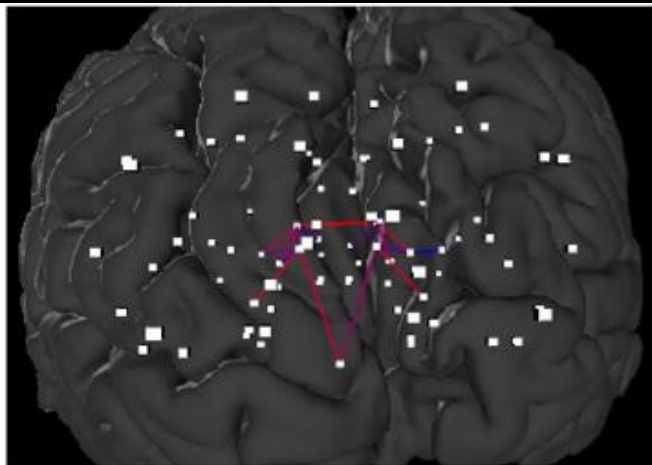
Consider the edges present in at least

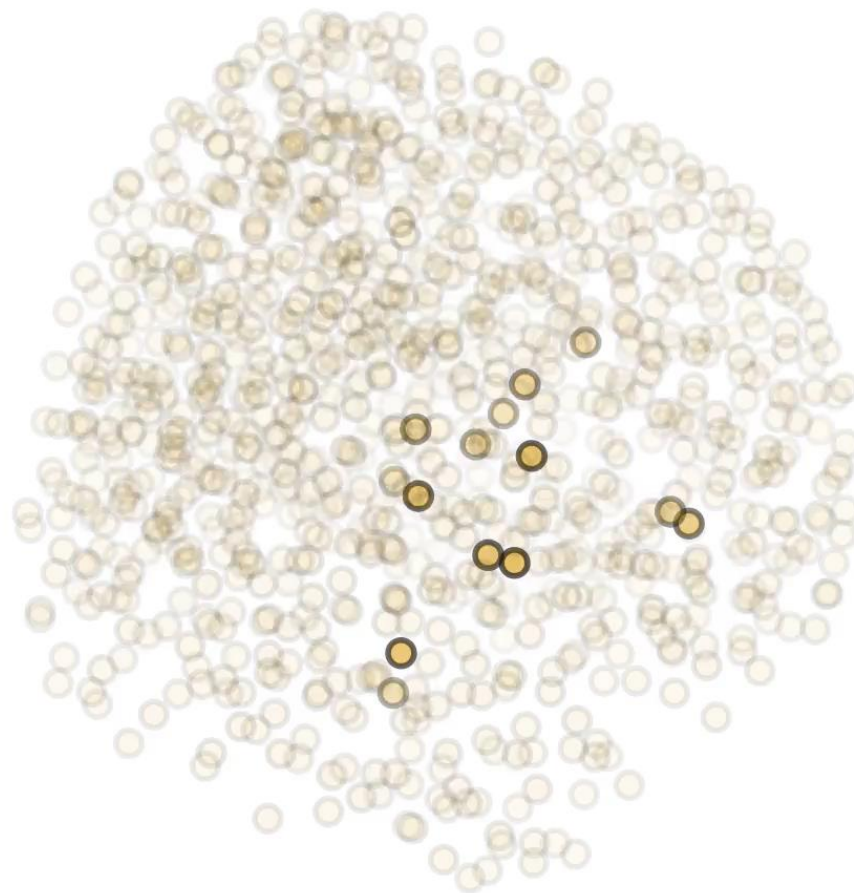
- $K=477$  braingraphs
  - $K=476$  braingraphs
  - .
  - $K=2$  braingraphs
  - $K=1$  braingraphs
- .. More and more edges appear (naturally).

**Growing tree-like structure!!! (surprising).**



Observed by Csaba Kerepesi







# The Consensus Connectome Dynamics

- By our assumption, this growing structure describes the axon-development of the human brain (first in the literature)
- The growing structure is robust (independent of the specifically chosen dataset)
- We can assign DIRECTIONS to the graph edges by using CCD (first in the literature);
- These directions are robust.

# Further information, references, on-line papers, databases, webserver

- <https://pitgroup.org>
- <http://grolmusz.pitgroup.org>
- Braingraphs: <http://braingraph.org>

Thank you very much!

